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The D^- centre in a quantum well in the presence of parallel electric and strong magnetic fields

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Abstract

An analytical approach to the problem of a negatively charged donor in an infinitely deep quantum well (QW) in the presence of parallel electric and strong magnetic external fields both directed perpendicular to the heteroplanes is developed. The double adiabatic approximation is employed. The dependences of the binding energy on the field strengths, the width of the well and the position of the impurity within the well are derived in explicit form. The effect of the inversion of the electric field is investigated. It is shown that the combined potential acting on the 'outer' electron resembles that of a double QW. When the levels associated with the two effective QWs anticross, a resonant structure arises. The explicit dependence of the resonant splitting on the width of the QW, the strength of the electric field and the position of the impurity are obtained. Using the parameters associated with the GaAs QW, estimates of the inversion shift of the binding energy and the frequency of the emitted resonant radiation induced by the electric field are made.

1. Introduction

During the last decade, the subject of a negatively charged donor (D^-) in quasi-two-dimensional systems has been studied extensively both experimentally and theoretically. Much of this work has concentrated on GaAs/GaAlAs structures; particularly on isolated single quantum wells (QWs) subjected to an external magnetic field directed perpendicular to the heteroplanes. The reason for this is that the magnetic field significantly increases the stability of the charged donor. A comprehensive study and summary of the problem of the H^- in the presence of a strong magnetic field in an unbound medium was given recently by Al-Hujaj and Schmelcher in [1]. Since the D^- centres were identified in confined systems in magneto-optical

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spectra by Huant *et al* [2] a considerable literature of theoretical work has arisen (see [3, 4] and references therein). The primary object was to calculate the binding energy of the D^- donor as a function of the strength of the magnetic field directed perpendicular to the heteroplanes, the width of the QW and the position of the impurity centre within the QW. In particular it was found that the binding energy increases with the magnetic field strength.

Although studies of charged complexes including the D^- in the QW in the presence of an electric field have been completed [5, 6], this problem is not an ideal candidate for experimental investigation. One of the reasons for this is that the binding energy of the D^- is small. The binding energy $|W_0|$ of the D^- in bulk material was calculated in [7, 8] yielding $|W_0| = 0.055R_y$ where R_y is the Rydberg constant of the neutral donor. The relevant electric field providing a significant decay of the system is defined by $F_0 = (2\mu)^{1/2}|W_0|^{3/2}(\hbar e)^{-1}$ where μ is the electron effective mass. Because of the small binding energy $|W_0|$, the D^- in bulk material decays significantly in the presence of even rather weak electric fields, i.e. for fields of a typical strength $F_0 = 7.8\,\mathrm{kV\ m^{-1}}$ for the GaAs semiconductor. The fields F applied to QWs of standard widths are of the same strengths. Thus D^- in an ordinary QW subjected to relatively weak electric fields tends to be unstable.

In the presence of a strong magnetic field the situation is completely different. Since the strong magnetic field leads to a significant increase in the binding energy of the D^- it is expected to be much more stable with respect to effects of the electric field. Clearly the study of the D^- in QW structures is important and the optical and transport properties of devices made from these structures are strongly affected by the presence of external fields. Recently Grill and Dohler [9] found that a charged donor has a strong impact on the transport properties of a two-dimensional electron gas in δ-doped heterostructures. A study of the neutral donor in the QW in the presence of parallel electric and magnetic fields is in progress. Latge *et al* [10] calculated the intradonor optical absorption spectra. The inversion effect of the electric field and the coherent resonant tunnelling in the impurity QW have been studied in [11] and [12] respectively. In contrast to this the problem of the D^- in the QW subjected to parallel electric and magnetic fields has not been addressed in the literature.

The majority of theoretical papers on the problem of the charged donor are based on numerical calculations, which usually rely upon a variational method. However, a numerical approach typically requires substantial computational efforts. Analytical results are of immediate interest because the basic physics of the problem remains transparent throughout the analysis. In addition a combination of analytical and numerical methods improves the accuracy of the calculation of the impurity states in the QWs [13]. The present paper provides an analytical investigation of the combined novel effects provided by the confinement and the external fields on the negatively charged donor. The latter is placed in an infinitely deep QW in the presence of parallel electric and strong magnetic fields both directed perpendicular to the heteroplanes. The effect of the magnetic field is taken to be much greater than that of the Coulomb field of the impurity centre. Note that extremely strong magnetic fields, in excess of 60 T, are nowadays available experimentally [14]. The extra confinement caused by a magnetic field renders the impurity states less sensitive to the form of the barrier potential. In the present work we consider a reasonably wide QW for which the width is greater than the impurity Bohr radius. Thus the approximation of an infinitely deep QW applied here is justified. The impurity can be positioned anywhere within the QW. The dependences of the binding energy of the D^- upon the field strengths, the width of the QW and the position of the impurity within the well are found in explicit form. Specific effects of the confined structure induced by the inversion of the electric field are studied. Resonant states of the extra electron caused by the confinement and the electric field are shown to occur. Using the parameters associated with the GaAs QW, estimates of the values to be expected in an experiment are made.

The paper is organized as follows. In section 2 the details of the analytical approach are presented and the general equation is derived. In sections 3 and 4 the limiting cases of weak and strong electric field, particularly the effect of the inversion of the electric field, are considered, respectively. The resonant tunnelling of the extra electron is studied in section 5. We provide a discussion of our results in section 6 and the conclusions in section 7.

2. General theory

The z-axis is chosen to point along the direction of the uniform magnetic ($\boldsymbol{B}$) and electric ($\boldsymbol{F}$) fields, which are applied perpendicular to the heteroplanes. The QW is treated as a square well of width d bounded by infinite barriers at the planes $z = \pm d/2$. The parameters relevant to the calculation are the impurity Bohr radius (a_0), the magnetic length (a_B) and the distance of the impurity centre (b) from the mid-point of the QW that is taken to be at $z = 0$. They are defined as usual by

$$a_0 = \frac{4\pi\varepsilon\varepsilon_0\hbar^2}{\mu e^2} \qquad a_B = \sqrt{\frac{\hbar}{eB}}$$

where ε is the dielectric constant and μ is the electron effective mass. We take the energy bands to be parabolic, nondegenerate and separated by a wide energy gap.

In the effective mass approximation the equation describing the D^- donor formed by the impurity centre and two spinless electrons at positions $\boldsymbol{r}_i(\boldsymbol{\rho}_i, z_i)$ ($i = 1, 2$) has the form

$$\left\{\sum_{i=1,2}\left[\frac{1}{2\mu}\left(-\mathrm{i}\hbar\boldsymbol{\nabla}_i + \frac{e}{2}[\boldsymbol{B}\boldsymbol{r}_i]\right)^2 - \frac{e^2}{4\pi\varepsilon_0\varepsilon|\boldsymbol{r}_i - b\boldsymbol{e}_z|} - eF(z_i - b)\right] + \frac{e^2}{4\pi\varepsilon_0\varepsilon|\boldsymbol{r}_2 - \boldsymbol{r}_1|}\right\} \times \Psi(\boldsymbol{r}_1, \boldsymbol{r}_2) = E\Psi(\boldsymbol{r}_1, \boldsymbol{r}_2) \tag{2.1}$$

where $\boldsymbol{e}_z$ is the unit vector. We omit the spin Zeeman term because it does not influence the binding energy of D^- that is the subject of our investigation.

By solving this equation subject to the boundary conditions

$$\Psi\left(\boldsymbol{\rho}_1, \pm\frac{d}{2}; \boldsymbol{\rho}_2, \pm\frac{d}{2}\right) = 0, \tag{2.2}$$

the energy E and wavefunction Ψ can be found in principle. In the limit of strong magnetic field for which

$$a_B/a_0 \ll 1 \tag{2.3}$$

the adiabatic approximation becomes appropriate. The electronic motion parallel and perpendicular to the magnetic field are separated. The solution to equation (2.1) describing the spin-singlet state of the D^- is taken in the form

$$\Psi(\boldsymbol{r}_1, \boldsymbol{r}_2) = \frac{1}{\sqrt{2}}\chi_\perp(\boldsymbol{\rho}_1)\chi_\perp(\boldsymbol{\rho}_2)[f(z_1)\varphi(z_2) + f(z_2)\varphi(z_1)]. \tag{2.4}$$

In the above expression the function

$$\chi_\perp(\boldsymbol{\rho}) = \frac{1}{\sqrt{2\pi}a_B}\exp\left(-\frac{\rho^2}{4a_B^2}\right) \tag{2.5}$$

describes the transverse motion of the electron with the energy $E_\perp = \frac{\hbar eB}{2\mu}$ (ground Landau level) in the x–y plane. The function $f(z)$ is the ground state wavefunction of the 'inner' electron tightly bound by the impurity centre (D^0 state). The function $f(z)$ satisfies

$$-\frac{\hbar^2}{2\mu}\frac{\mathrm{d}^2 f(z)}{\mathrm{d}z^2} + [V(z) - eF(z - b)]f(z) = \Lambda f(z) \tag{2.6}$$

with the boundary conditions

$$f(\pm d/2) = 0 \tag{2.7}$$

and

$$V(z) = -\frac{e^2}{4\pi\varepsilon_0\varepsilon}\int \frac{|\chi_\perp(\boldsymbol{\rho})|^2}{\sqrt{\rho^2+(z-b)^2}}\,\mathrm{d}\boldsymbol{\rho}. \tag{2.8}$$

It is convenient to introduce the notation $\Lambda = -R_y/\lambda^2$ where $R_y = e^2/8\pi\varepsilon_0\varepsilon a_0$ is the impurity Rydberg constant and λ (<1) is the corresponding quantum number. We consider a reasonably wide QW for which $d > a_0$ such that the ground state has a quasi-Coulomb character and the energy $\Lambda < 0$ [15]. Narrowing the QW increases the binding energy of the D^- which makes it more favourable for an experimental study. However, the effects caused by the electric field F and shift b of the impurity within the QW become insignificant, because of the small energy eFd and displacement $|b| \leqslant d/2$ for the QW of small width d. For the subject of this paper, i.e. study of the dependences of the binding energy on the electric field and displacements, relatively wide QWs are preferable. Also we assume that the impurity centre is separated from the edge of the QW by a distance greater than the effective radius of the ground state which in turn implies that $d/2 - |b| \gg a_0\lambda$. For this case it was shown in [15] that the effect of the boundaries of the QW on the electronic states is exponentially small. For the GaAs QW of width $d = 320$ Å subjected to a magnetic field $B = 30$ T for which the numerical calculations are performed, the allowed displacements b are limited to being of the order of 100 Å. This shift is more than 60% of half of the width of the QW. For such displacements the relative change of the electron density induced by the boundary is less than 0.1. As the impurity centre is approaching the boundary of the QW ($|b| \approx d/2$), the longitudinal states of the electrons are changed drastically. The impurity positioned at the interface requires special consideration that is beyond the scope of this paper.

Further, we use the solution to equation (2.6) for the bulk semiconductor ($d = \infty$) obtained originally in [16]. The expression for the wavefunction $f(z)$ is given by

$$f(z) = \frac{1}{\sqrt{a_0\lambda}}\exp\left(-\frac{|z-b|}{a_0\lambda}\right). \tag{2.9}$$

The quantum number λ (<1) is the smallest root of the equation

$$-\frac{1}{2}\ln\left(\frac{\gamma\lambda^2}{2}\right) + \psi(1-\lambda) + \frac{1}{2\lambda} + \frac{3}{2}C = 0 \tag{2.10}$$

where $\gamma = a_0^2/a_B^2$ ($\gg 1$), C is the Euler constant ($=0.577$) and $\psi(x)$ is the psi function (the logarithmic derivative of the gamma function). In the logarithmic approximation for which $\gamma \gg 1$ and $\ln\gamma \gg 1$, we have

$$\lambda = \left(\ln\frac{\gamma}{2} - C\right)^{-1} \ll 1. \tag{2.11}$$

At this stage we use a fact that is unique to the one-dimensional quasi-Coulomb potential $V(z)$, equation (2.8): the energy Λ and the wavefunction $f(z)$ of the ground state are described by the quantum number $\lambda < 1$. This means that the effective motion of the electron in the ground state is the fastest possible. This is the basis for an adiabatic approximation according to which the wavefunction of the 'outer' electron $\varphi(z)$ is governed by the effective potential representing an average with respect to the function $f(z)$. The equation for the wavefunction $\varphi(z)$ has the form

$$-\frac{\hbar^2}{2\mu}\frac{\mathrm{d}^2\varphi(z)}{\mathrm{d}z^2} + [U(z) - eF(z-b)]\varphi(z) = W\varphi(z) \tag{2.12}$$

with

$$U(z) = -\frac{e^2}{4\pi\varepsilon_0\varepsilon}\left(\int \frac{|\chi_\perp(\boldsymbol{\rho})|^2}{\sqrt{\rho^2+(z-b)^2}}\,\mathrm{d}\boldsymbol{\rho} - \int \frac{|\chi_\perp(\boldsymbol{\rho}_1)|^2|\chi_\perp(\boldsymbol{\rho}_2)|^2|f(z')|^2}{\sqrt{(\boldsymbol{\rho}_1-\boldsymbol{\rho}_2)^2+(z-z')^2}}\,\mathrm{d}\boldsymbol{\rho}_1\,\mathrm{d}\boldsymbol{\rho}_2\,\mathrm{d}z'\right) \tag{2.13}$$

and with

$$W = E - 2E_\perp - \Lambda + \frac{5\hbar^2e^2F^2}{32\mu\Lambda^2}. \tag{2.14}$$

The last term on the right-hand side of equation (2.14) is the correction to the energy $\Lambda = -R_y/\lambda^2$ induced by the electric field F. The wavefunction $\varphi(z)$ satisfies the boundary conditions

$$\varphi(\pm d/2) = 0. \tag{2.15}$$

Below, we focus on the binding energy of the D$^-$. Although the additional spin Zeeman term does not affect the binding energy, it influences the total energy of the D$^-$. It was pointed out in [1] that for the hydrogen negative ion H$^-$ subjected to very weak magnetic fields, the spin-singlet state is the ground state of the system. With increasing magnetic field strength the spin-triplet state becomes the ground state. This is caused by the spin Zeeman term, which is determined by the g-factor of the free electron, $g_0 = 2$, and which lowers the total energy of the spin-triplet state. The crossover takes place at $B_1 \approx 10^4$ T, which is much less than the atomic unit of the magnetic field $B_0 = 2.35 \times 10^5$ T. It is important that the spin-triplet state, being the ground state for $B > B_1$, is considerably less bound than the spin-singlet state. For strong magnetic fields $B > B_0$, the binding energy of the spin-singlet state exceeds that of the spin-triplet state by a factor of more than 2.

For GaAs bulk material having the electron effective g-factor $g_e \approx -0.44$, the spin Zeeman shift of the total energy of the spin-triplet state is less than that of the H$^-$. The crossover of the spin-singlet and spin-triplet states takes place at $B_1 \approx 3$ T, which is less than the 'donor' magnetic field $B_0 = 6$ T, determined by the condition $a_0 = a_B$. Thus in the presence of strong magnetic fields $B > B_0 > B_1$, the spin-singlet state being the excited state remains the most bound state of the D$^-$ in GaAs bulk crystal.

In the QW the binding energy of the D$^-$ depends strongly on the position of the impurity centre within the well. This effect can be more important than the shift of the total energy of the D$^-$ caused by the spin Zeeman term. For the impurity centre positioned close to the mid-point of the wide QW, the results are qualitatively the same as for the bulk material. However, for the impurity centre displaced from the mid-point of the QW the situation is different. It was shown in [4] that in the presence of a magnetic field $B \approx 60$ T for the impurity centre shifted to $b < b_0 = 0.25d$ from the mid-point of the GaAs QW of width $d = 200$ Å, the binding energy of the spin-singlet state exceeds that of the spin-triplet state, while for the displacements $b > b_0$, the spin-triplet state becomes more bound. The narrower the QW, the less the critical displacement b_0. In the following we consider sufficiently wide QWs subjected to strong magnetic fields for which the spin-singlet state remains the most tightly bound state for a relatively wide region $b < b_0$.

The binding energy E_b of the D$^-$ in the QW is defined as usual by the difference between the sum of the energies of the neutral donor and the electron in the QW $\left(E_\perp + \Lambda - \frac{5\hbar^2e^2F^2}{32\mu\Lambda^2}\right) + (E_\perp + E^{(0)})$ and the energy of the charged donor E. The energy $E^{(0)}$ is the energy of the ground state of the electron in the QW in the presence of the electric field. Using (2.14) we have

$$E_b = E^{(0)} - W. \tag{2.16}$$

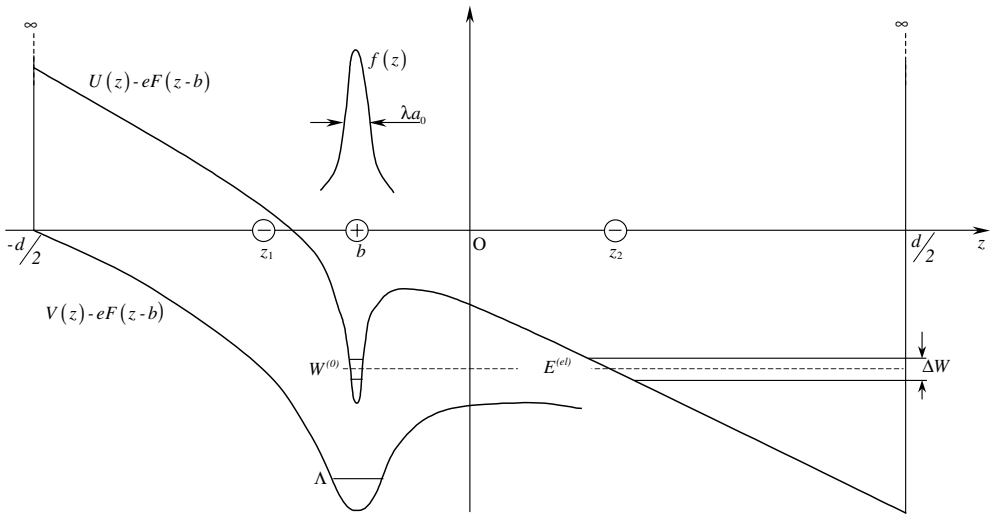

Figure 1. A sketch of the potentials $V(z)-eF(z-b)$ (equation (2.8)) and $U(z)-eF(z-b)$ (equation (2.13)) and energies of the 'inner' electron $\Lambda = -R_y/\lambda^2$ and 'outer' electron, for which $W^{(0)}$ (equation (5.4)) is the donor level, $E^{(el)} = E^{(0)}$ (equation (4.2)) is the 'electric' level and ΔW (equation (5.8)) is the resonance gap. The impurity centre is positioned at $z = b$ in the QW of width d subjected to electric field F. The function $f(z)$ (equation (2.9)) is the wavefunction of the 'inner' electron.

A sketch of the effective potentials $V(z) - eF(z - b)$ (equation (2.8)) and $U(z) - eF(z - b)$ (equation (2.13)), the resulting wavefunction $f(z)$ (equation (2.9)) and the energies Λ in (2.6) and W in (2.12) is shown in figure 1.

Under the condition $\lambda < 1$ the effective radius of the ground state $a_0\lambda$ is less than the Bohr radius a_0. This is used in proceeding with the analytical calculations. In the zeroth approximation for (2.13) we arrive at

$$U(z) \approx \begin{cases} -\dfrac{e^2\sqrt{\pi}(\sqrt{2}-1)}{8\pi\varepsilon_0\varepsilon a_B} & \text{at } z = b \\ -\dfrac{e^2 a_B^2}{4\pi\varepsilon_0\varepsilon|z-b|^3} & \text{for } |z-b| \gg a_B \end{cases} \tag{2.17}$$

and the potential $U(z)$ has the form of a finite well of the effective width a_B. Since the magnetic length a_B is much less than the Bohr radius a_0, we replace the potential $U(z)$ (equation (2.13)) by a δ-function-type potential. Leaving aside the effect of the boundaries of the QW, we put

$$U(z) = -\frac{\hbar^2 q_0}{\mu}\delta(z-b) \tag{2.18}$$

where the parameter q_0 is defined by the ground energy level W_0 in the bulk semiconductor, namely $q_0 = (-2\mu W_0/\hbar^2)^{1/2}$.

Further, we consider a wide QW for which the energies W are negative ($W < 0$). The solution to equation (2.12) satisfying the boundary conditions (2.15) is given by

$$\varphi(z) = -\int_{-d/2}^{+d/2} G_W(z,z')U(z')\varphi(z')\,\mathrm{d}z' \tag{2.19}$$

where the Green function $G_W(z,z')$ of the electron in the rectangular QW of width d in the presence of an electric field F has the form

$$G_W(z,z') = \frac{2\pi\mu}{\hbar^2\sigma}\{[\mathrm{Bi}(-\eta')\,\mathrm{Ai}(-\eta_1) - \mathrm{Ai}(-\eta')\,\mathrm{Bi}(-\eta_1)] \times [\mathrm{Bi}(-\eta_2)\,\mathrm{Ai}(-\eta) - \mathrm{Ai}(-\eta_2)\,\mathrm{Bi}(-\eta)]\} \times \{[\mathrm{Bi}(-\eta_1)\,\mathrm{Ai}(-\eta_2) - \mathrm{Ai}(-\eta_1)\,\mathrm{Bi}(-\eta_2)]\}^{-1}; \qquad z > z' \tag{2.20}$$

and where $\mathrm{Ai}(u)$ and $\mathrm{Bi}(u)$ are the Airy functions [17]. In the above expression the following notations are used:

$$\sigma = \left(\frac{2\mu eF}{\hbar^2}\right)^{1/3}; \qquad \eta(z) = \sigma(z - b - z_0); \qquad z_0 = \frac{\hbar^2 q^2}{2\mu eF};$$

$$\eta' = \eta(z'); \qquad \eta_{1,2} = \eta\left(\mp\frac{d}{2}\right); \qquad q = \left(-\frac{2\mu W}{\hbar^2}\right)^{1/2}.$$

The expression for the Green function for the region $z < z'$ can be obtained from equation (2.20) by replacing z by z' and vice versa. The wavefunction $\varphi(z)$ and the Green function $G_W(z,z')$ both satisfy the boundary conditions (2.15).

Substituting the expressions (2.18) and (2.20) into equation (2.9) and then setting $z = b$ in (2.19), we obtain the transcendental equation

$$\frac{2\pi q_0}{\sigma}\frac{[\mathrm{Bi}(-\eta_0)\,\mathrm{Ai}(-\eta_1) - \mathrm{Ai}(-\eta_0)\,\mathrm{Bi}(-\eta_1)][\mathrm{Bi}(-\eta_2)\,\mathrm{Ai}(-\eta_0) - \mathrm{Ai}(-\eta_2)\,\mathrm{Bi}(-\eta_0)]}{[\mathrm{Bi}(-\eta_1)\,\mathrm{Ai}(-\eta_2) - \mathrm{Ai}(-\eta_1)\,\mathrm{Bi}(-\eta_2)]} = 1 \tag{2.21}$$

where $\eta_0 = -\sigma z_0$. By solving equation (2.21) the parameter q and the energy W can be found as functions of the width d of the QW, the position of the impurity centre b and the electric field strength F.

In order to study qualitatively the dependence of the energy W_0 and the parameter q_0 on the magnetic field we solve equation (2.12) with the potential $U(z)$ (equation (2.13)) for the bulk semiconductor ($d = \infty$) in the absence of the electric field ($F = 0$). It is convenient to introduce the notation

$$\nu = \sqrt{-\frac{R}{W_0}}; \qquad u = \frac{2(z-b)}{a_0\nu}; \qquad g_0 = \frac{2\rho}{a_0\nu}; \qquad g_{12} = \frac{2|\boldsymbol{\rho}_1 - \boldsymbol{\rho}_2|}{a_0\nu}.$$

Equation (2.12) then becomes

$$\varphi''(u) + \lambda[\langle 0|(u^2 + g_0^2)^{-1/2}|0\rangle - \langle 1,2|(u^2 + g_{12}^2)^{-1/2}|1,2\rangle]\varphi(u) - \tfrac{1}{4}\varphi(u) = 0 \tag{2.22}$$

where $\langle 0||0\rangle$ and $\langle 1,2||1,2\rangle$ are averages with respect to the functions $\chi_\perp(\boldsymbol{\rho})$ and $\chi_\perp(\boldsymbol{\rho}_1)\chi_\perp(\boldsymbol{\rho}_2)$ respectively. Setting in (2.22) $u \gg g_0, g_{12} \sim 2a_B/a_0\nu$ and then neglecting the term proportional to $|u|^{-3}$ we obtain

$$\varphi(u) = A\exp\left(-\tfrac{1}{2}u\right) \tag{2.23}$$

where A is a constant.

In the region $u \ll 1$, an iteration is performed by double integration of equation (2.22) using the trial function $\varphi^{(0)}(u)$ satisfying the boundary conditions

$$\varphi^{(0)}(0) = B; \qquad \varphi^{(0)'}(0) = 0$$

where B is a constant.

A comparison of the coefficients is then performed for the result of the integration and the expansion of the function (2.23) for $u \ll 1$. When terms of the same order are equated a set of linear algebraic equations are found. The system of these equations is solved by the determinantal method to give the following expression for the quantum number ν that in turn leads to the binding energy $|W_0|$ of the D$^-$ in bulk semiconductors:

$$\frac{|W_0|}{R_y} \approx \left(1 + \frac{\sqrt{2\pi}(\sqrt{2}-1)}{\sqrt{\gamma}}\right)^{-2}. \tag{2.24}$$

It follows from equation (2.24) that for increasing magnetic field strength, the binding energy of the D^- both in the bulk semiconductor and in the QW increases. This dependence coincides qualitatively with those obtained numerically by a variational approach (see [1, 4] and references therein). Note that for strong laboratory magnetic fields $B \approx 30$–60 T corresponding to the parameter $\gamma \approx 5$–10 for the GaAs material, equation (2.24) gives for the ratio $|W_0(\gamma)|/|W_0(5)|$ the values 1.14 and 1.21 for the parameters $\gamma = 8$ and 10 respectively. This is close to the relative energies 1.16 and 1.25 respectively presented in [1]. If a finite height of the barriers bounding the QW is taken into account [18], the binding energy of the D^- for the parameter $\gamma = 5$ in the QW of width $d = 4\ a_0$ calculated from equation (2.21), coincides to an accuracy of several per cent with the result obtained numerically [3]. Thus our approach to the description of the D^- in the presence of strong magnetic fields is qualitatively justified. A further quantitative study requires the numerical solution of equation (2.12) in which the potential $U(z)$ should be calculated using the wavefunction $f(z')$ (equation (2.9)).

Note that the binding energy in the bulk semiconductors was studied in detail in a number of papers for a wide range of the magnetic field strength [1]. This allows one to treat the value q_0 as a parameter of the problem being given either by experiments or numerical calculations. Below we focus on the dependence of the binding energy E_b on the width d of the QW, the displacement of the impurity centre b and the electric field strength F.

3. Weak electric field

For a weak electric field $F \ll F_0$ where $F_0(q) = \frac{(2\mu)^{1/2}|W(q)|^{3/2}}{e\hbar}$ is the effective electric field associated with the charged donor in the QW, an asymptotic expression for the Airy functions $\mathrm{Ai}(u)$ and $\mathrm{Bi}(u)$ for $u < 0, |u| \gg 1$ in equation (2.21) can be used [17]. Dropping some cumbersome algebra, equation (2.21) is transformed into

$$\frac{2\alpha_0}{\alpha}\frac{\sinh\frac{\alpha(1-s)}{2}\sinh\frac{\alpha(1+s)}{2}}{\sinh\alpha}\left\{1+\frac{1}{4}\left(\frac{F}{F_0}\right)\alpha^2\left[s\coth\alpha+\frac{(1-s)^2}{4}\coth\frac{\alpha(1-s)}{2}\right.\right.$$
$$\left.-\frac{(1+s)^2}{4}\coth\frac{\alpha(1+s)}{2}\right]+\frac{5}{32}\left(\frac{F}{F_0}\right)^2\left[1+\alpha\left(\coth\alpha\right.\right.$$
$$\left.\left.\left.-\frac{(1-s)}{2}\coth\frac{\alpha(1-s)}{2}-\frac{(1+s)}{2}\coth\frac{\alpha(1+s)}{2}\right)\right]\right\}=1 \tag{3.1}$$

where $\alpha_0 = q_0 d$, $\alpha = qd$ and where $s = 2b/d$.

For small displacements of the impurity from the mid-point of the QW ($b = 0$) for which $s \ll 1$, the explicit solution to equation (3.1) can be found to give in turn for the energy W

$$W(F, b) = W(0, 0) + \Delta W(F, b) \tag{3.2}$$

where $W(0, 0)$ is the energy of the 'outer' electron for the case of the impurity positioned at the centre of the QW ($b = 0$) of width d in the absence of the electric field ($F = 0$) and where $\Delta W(F, b)$ is the shift of the energy induced by the electric field F and the displacement of the impurity $b = ds/2$. The energy $W(0, 0)$ is defined by the expression $W(0, 0) = -\frac{\hbar^2 q_1^2}{2\mu}$ in which the parameter q_1 satisfies the equation

$$\frac{\alpha_0}{\alpha_1}\tanh\frac{\alpha_1}{2} = 1; \qquad \alpha_1 = q_1 d. \tag{3.3}$$

The correction to the energy ΔW can be written in the form

$$\Delta W(F, b) = -|W(0, 0)|\frac{5}{16F_0^2}[(F - F_1)^2 - \Phi(\alpha_1)F_1^2] \tag{3.4}$$

with

$$\Phi(\alpha) = 1 + 10\frac{(1 - \frac{\alpha}{\sinh\alpha})\cosh^2\frac{\alpha}{2}}{\alpha^2(\frac{\alpha}{2}\coth\frac{\alpha}{2} - 1)^2} \tag{3.5}$$

and where

$$F_1(\alpha_1, s) = -\frac{4}{5}F_0\frac{\alpha_1^2(\frac{\alpha_1}{2}\coth\frac{\alpha_1}{2} - 1)}{(\sinh\alpha_1 - \alpha_1)}s \tag{3.6}$$

is the effective electric field caused by the displacement of the impurity s from the centre of the QW of finite width d.

It follows from equation (3.4) that in the absence of the electric field ($F = 0$) the displacement of the impurity from the centre of the QW leads to an increase in energy $\Delta W(0, b)$ where

$$\Delta W(0, b) = |W(0, 0)|\frac{5}{16}[\Phi(\alpha_1) - 1]\left(\frac{F_1}{F_0}\right)^2 > 0. \tag{3.7}$$

The expression (3.7) coincides with that obtained in [18] in which the double adiabatic approximation was applied originally and in which the D^- in the QW subjected to a strong magnetic field has been considered. From equation (2.16) it is clear that the binding energy E_b decreases with increasing shift of the impurity b. This result is common to the neutral [19] and charged [4] donors both in the absence and in the presence of the magnetic field.

Clearly equation (3.4) shows that the electric field F shifts the energy of the outer electron bound by the impurity located at the centre of the QW ($b = 0$) towards lower energies. The corresponding red-shift $\Delta W(F, 0)$ is defined by

$$\Delta W(F, 0) = -|W(0, 0)|\frac{5}{16}\left(\frac{F}{F_0}\right)^2 < 0. \tag{3.8}$$

For the electric fields $F_{+,-} = F_1[1 \pm \Phi(\alpha_1)^{1/2}]$, the energy shift $\Delta W(F_{+,-}, b) = 0$. For these electric fields the blue- and red-shifts induced by the displacement b and the electric field F, respectively, cancel. For the region of the electric fields $F_- < F < F_+$ the effect of the displacement b dominates over that of the electric field F and $\Delta W(F, b) > 0$ with a maximum

$$\Delta W_{max} = |W(0, 0)|\frac{5}{16}\Phi(\alpha_1)\left(\frac{F_1}{F_0}\right)^2; \qquad \text{at } F = F_1. \tag{3.9}$$

For the electric fields $F < F_-$ and $F > F_+$, the energy shift $\Delta W(F, b) < 0$. The binding energy E_b can be obtained from equation (2.16) in which the energy W is defined by equations (3.2) and (3.4). The energy of the ground state of the electron in the QW in the presence of a weak electric field $E(0)$ has the form [20]

$$E^{(0)} = \frac{\hbar^2\pi^2}{2\mu d^2} - \frac{1}{24\pi}\left(\frac{15}{\pi^2} - 1\right)\frac{\mu e^2F^2d^4}{\hbar^2}. \tag{3.10}$$

The binding energy E_b as a function of the weak electric field F for different small displacements b is shown in figure 2. The dependence of the binding energy E_b on small displacements of the impurity b for different weak electric field strengths F is depicted in figure 3.

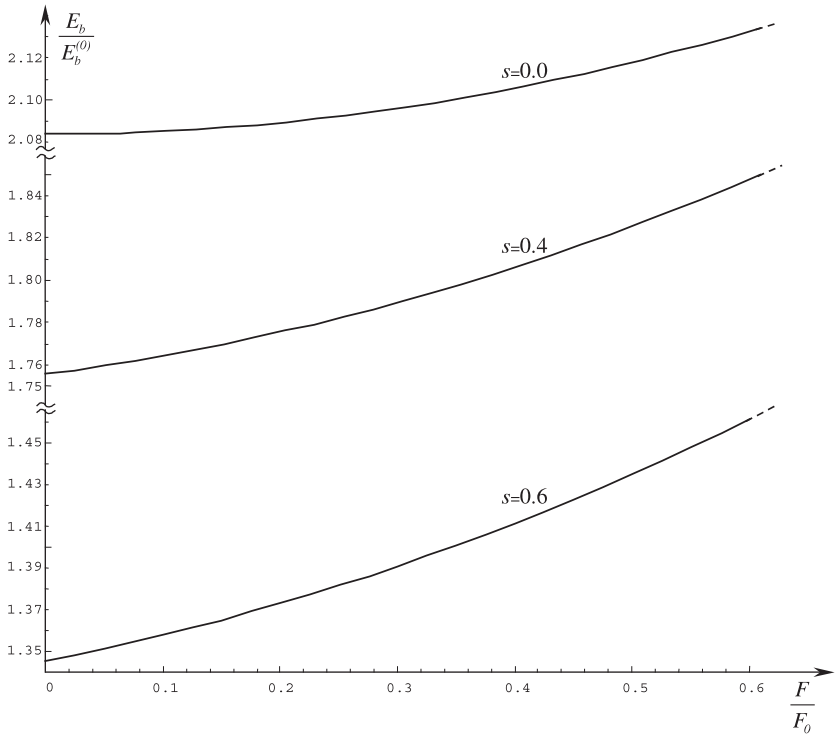

Figure 2. The dependence of the binding energy $E_b/E_b^{(0)}$ ($E_b^{(0)}$ is the binding energy in bulk material) of the D^- in a GaAs QW of width $d = 320$ Å calculated from equations (2.16), (3.2) and (3.4) on the dimensionless electric field F/F_0 (F_0 is the donor field) for different displacements of the impurity $b = s(d/2)$.

3.1. Inversion effect of the electric field

It follows from equation (2.21) that the QW containing the D^- is suitable for demonstrating the inversion effect of the electric field, i.e. changes of the energy of the 'outer' electron $W(F, b)$ with reversion of the direction of the electric field ($+F \to -F$). In particular, for weak electric fields $F \ll F_0$ and small displacements $s \ll 1$, the inversion energy shift $\delta W = W(+F, b) - W(-F, b)$ can be derived from equation (3.4) with the result

$$\delta E_b = -\delta W = \frac{5}{4}|W(0,0)|\frac{F_1 F}{F_0^2}. \tag{3.11}$$

The inversion shift $\delta W \sim F F_1 \sim Fb$ increases with increasing electric field F and displacement b and vanishes when the impurity centre is positioned at the mid-point of the QW ($b = 0$). The wider the QW, the less the effects, including the inversion shift (3.11) provided by the displacement b. It follows from (3.3), (3.5), (3.6) that for a wide QW for which $\alpha_0 = q_0 d \gg 1$,

$$W(0,0) \approx W_0[1 - 4\exp(-\alpha_0)]; \qquad F_1 \approx -\tfrac{4}{5}F_0 s\alpha_0^3 \exp(-\alpha_0);$$
$$\Phi(\alpha_0) = 10\alpha_0^{-4}\exp(\alpha_0)$$

and in the limiting case of the bulk material ($\alpha_0 \to \infty$), $W(0,0) = W_0$, $F_1 = 0$, $\delta E_b = 0$. As expected, in this case the expression (3.8) coincides with that obtained earlier for the quasi-one-dimensional electron in the presence of an electric field in an unbounded medium [21].

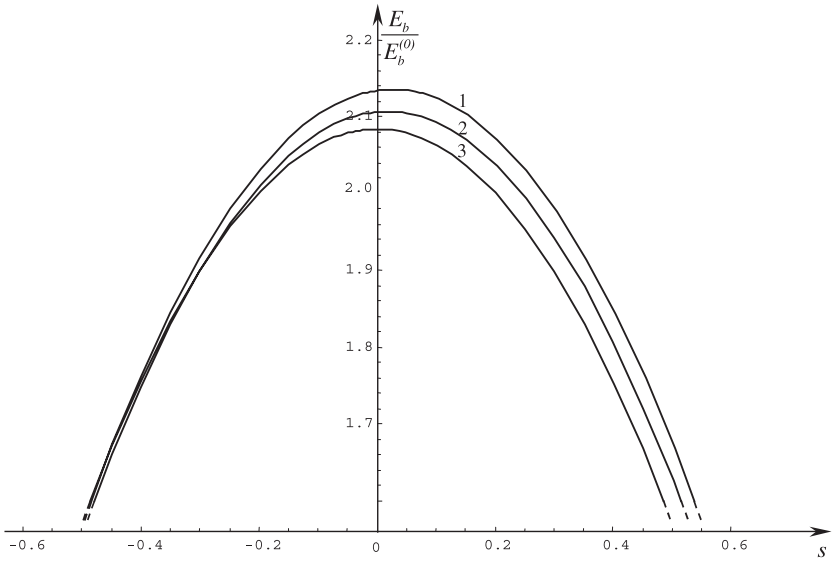

Figure 3. The binding energy E_b of the D$^-$ in a GaAs QW of width $d = 320$ Å calculated from equations (2.16), (3.2) and (3.4) and scaled to the binding energy $E_b^{(0)}$ in bulk material as a function of the dimensionless displacement of the impurity $s = 2b/d$ for the different electric field strengths $F/F_0 = 0.0$—(3); 0.4—(2); 0.6—(1), where F_0 is the donor field.

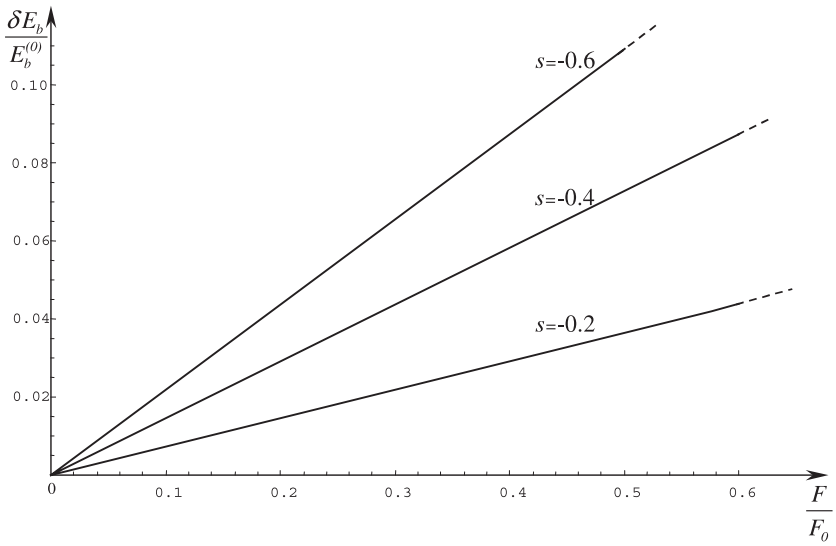

Figure 4. The dependence of the inversion shift of the binding energy $\delta E_b/E_b^{(0)}$ (equation (3.11); $E_b^{(0)}$ is the binding energy in bulk material) of the D$^-$ in a GaAs QW of width $d = 320$ Å on the dimensionless electric field F/F_0 (F_0 is the donor field) for different displacements of the impurity $b = s(d/2)$.

The dependence of the inversion shift of the binding energy $\delta E_b = E_b(+F, b) - E_b(-F, b)$ on the weak electric field strength F for different small displacements of the impurity b is shown in figure 4.

4. Strong electric field

In order to find the energy $E(0)$ determining the binding energy E_b in equation (2.16) we set in expression (2.21) $W \equiv E(0)$ and then in the zeroth approximation $q_0 = 0$. We have from equation (2.21)

$$\mathrm{Bi}(-\eta_1)\,\mathrm{Ai}(-\eta_2) - \mathrm{Bi}(-\eta_2)\,\mathrm{Ai}(-\eta_1) = 0. \tag{4.1}$$

For sufficiently strong electric field and wide QW an asymptotic expansion of the Airy functions $\mathrm{Ai}(-\eta_1)$ and $\mathrm{Bi}(-\eta_1)$ for $\eta_1 < 0$, $\mathrm{Bi}(-\eta_1) \neq 0$ can be employed and equation (4.1) reduces to $\mathrm{Ai}(-\eta_2) = 0$ having the smallest root $\eta_2 = 2.34$, giving in turn for the ground energy level $E^{(0)}$

$$E^{(0)} = -eF(d/2 - b) + 2.34E_0; \tag{4.2}$$

where $E_0(F) = \left(\frac{\hbar^2 e^2 F^2}{2\mu}\right)^{1/3}$.

Actually the energy $E^{(0)}$ in equation (4.2) is the ground energy level in the triangular well, having its bottom at the value $-eF\left(\frac{d}{2} - b\right)$ and formed by the electric field potential and right-hand boundary of the QW (see figure 1).

In the next approximation we take into account the effect of the impurity assuming the parameter q_0 in equation (2.21) to be small. Expanding the Airy functions $\mathrm{Ai}(u)$ and $\mathrm{Bi}(u)$ in equation (2.21) in the vicinity of the unperturbed value $E^{(0)}$ given by equation (4.2) and taking into account that $\mathrm{Bi}(-2.34)\,\mathrm{Ai}'^{-1}(-2.34) = -0.64$, we obtain for the energy $W = E^{(0)} + \Delta W$, where $E^{(0)}$ is defined by equation (4.2) and where the correction to the energy $E^{(0)}$ caused by the impurity has the form

$$\Delta W = -0.64(2\pi)E_0\left(\frac{F_0}{F}\right)^{1/3}\mathrm{Ai}^2(-\varsigma) \tag{4.3}$$

where $\zeta = -\sigma(d/2 - b) + 2.34$. The above equation determines the binding energy $E_b = E^{(0)} - W = -\Delta W$. It follows from equation (4.3) that the impurity potential leads to a red-shift of the energy $\Delta W < 0$, depending in particular on the position b of the impurity centre. This dependence is described by a factor $\mathrm{Ai}^2(-\zeta)$ in equation (4.3). For the impurity positioned at $b \approx d/2$ we have $\zeta \approx \eta_2$ and $E_b = \Delta W \approx 0$. When the impurity is shifted to the opposite boundary of the QW, the binding energy E_b increases and reaches a maximum at $\zeta = 1$, i.e. $b = d/2 - 1.34\sigma^{-1}$, with the result

$$E_{b,m} = 0.64 \times 0.536^2 \times 2\pi\left(\frac{F_0}{F}\right)^{1/3}E_0. \tag{4.4}$$

Upon further shift of the impurity in the same direction the binding energy E_b decreases and becomes exponentially small at $b \approx -d/2$. For these displacements of the impurity,

$$E_b = 0.32E_0\left(\frac{F_0}{F}\right)^{1/3}|\varsigma|^{-1/4}\exp\left(-\frac{4}{3}|\varsigma|^{3/2}\right) \tag{4.5}$$

where $|\zeta| = \sigma d - 2.34 \gg 1$. Expressions (4.2)–(4.6) are valid for a sufficiently strong electric field $F \gg F_0$ and wide QW. Figure 5 shows the binding energy E_b as a function of the displacement of the impurity b for different strong electric fields F.

For strong electric field F the inversion shift of the binding energy $\delta E_b = E_b(F, +b) - E_b(F, -b)$ becomes

$$\delta E_b = 0.64(2\pi)E_0\left(\frac{F_0}{F}\right)^{1/3}[\mathrm{Ai}^2(-\varsigma_+) - \mathrm{Ai}^2(-\varsigma_-)] \tag{4.6}$$

where $\varsigma_\pm = -\frac{\sigma d}{2}(1 \mp s) + 2.34$. The dependence of the inversion shift of the binding energy δE_b on the displacement of the impurity b for different electric field strengths F is depicted in figure 6.

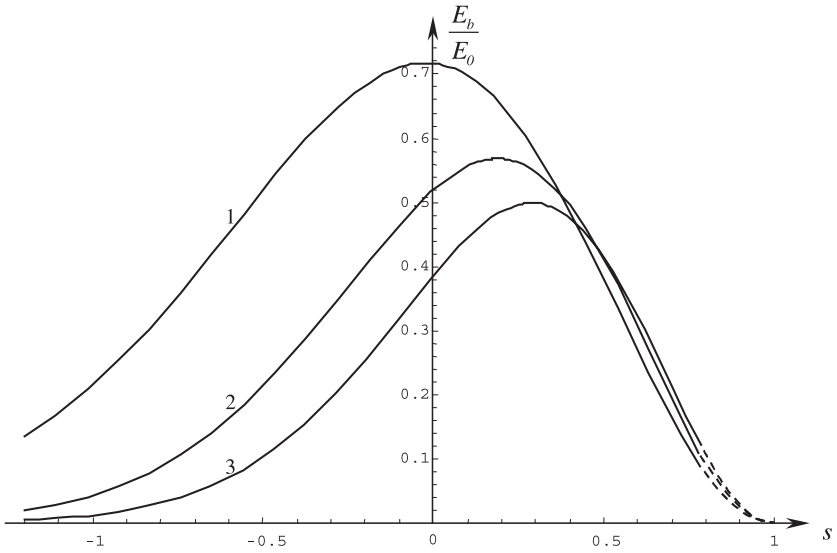

Figure 5. The binding energy $E_b = -\Delta W$ (equation (4.3)) of the D$^-$ in a GaAs QW of width $d = 320$ Å scaled to the energy $E_0 = \left(\frac{\hbar^2 e^2 F^2}{2\mu}\right)^{1/3}$ as a function of the dimensionless displacement of the impurity $s = 2b/d$ for the different electric field strengths $F/F_0 = 3.33$—(1); 6.66—(2); 10.0—(3), where F_0 is the donor field.

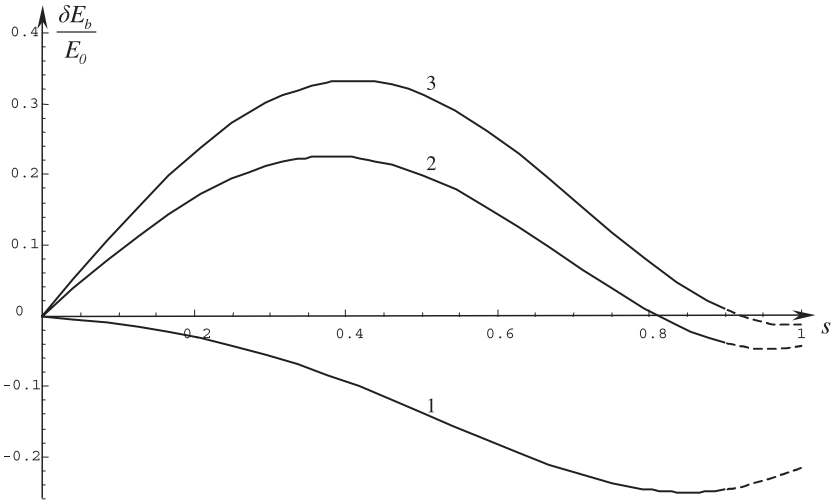

Figure 6. The dependence of the inversion shift of the binding energy $\delta E_b/E_0$ (equation (4.6); $E_0 = (\frac{\hbar^2 e^2 F^2}{2\mu})^{1/3}$) of the D$^-$ in a GaAs QW of width $d = 320$ Å on the dimensionless displacement of the impurity $s = 2b/d$ for the different electric field strengths $F/F_0 = 3.33$—(1); 6.66—(2); 10.0—(3), where F_0 is the donor field.

5. Coherent resonant tunnelling

It follows from figure 1 that the combined potential governing the states of the external electron has the appearance of a double QW. The first effective well is the so-called donor well, formed by the short-range potential and the left-hand boundary of the QW closest to the impurity. The second effective well (the triangular 'electric' well) is formed by the potential of the

electric field F and the right-hand boundary of the QW. When the levels associated with the two effective QWs anticross, the single QW containing the charged donor can be treated as a resonant structure. This structure is similar to that realized for the neutral donor [12].

For a weak electric field under the condition $|\eta_0| \gg 1$, equation (2.21) can be written in the form

$$\mathrm{Ai}(-\eta_2)\left\{1-\frac{q_0}{q}\left[1-\exp\left(-2q\left(\frac{d}{2}+b\right)\right)\right]\right\} + \frac{1}{2}\,\mathrm{Bi}(-\eta_2)\exp(-2\Phi)\times\left\{1-\exp\left[-2q\left(\frac{d}{2}+b\right)\right]\right\}=0 \tag{5.1}$$

where $\Phi=\frac{2}{3}|\eta_0|^{3/2}$.

The first term on the left-hand side of this equation describes the 'electric' levels (the first factor) and the donor level (the second factor). The last term is responsible for the tunnelling of the extra electron from the donor well towards the 'electric' well through the potential barrier with $\Phi \gg 1$.

Neglecting the tunnelling term as a zeroth approximation, equation (5.1) decomposes into two independent equations:

$$\mathrm{Ai}(-\eta_2)=0 \tag{5.2}$$

and

$$1-\frac{q_0}{q}\left\{1-\exp\left[-2q\left(\frac{d}{2}+b\right)\right]\right\}=0 \tag{5.3}$$

which arise from the two effective wells.

Equation (5.2) describes the 'electric' levels in the triangular well. The solution to this equation, corresponding to the ground level, has the form $\eta_2 = 2.34$. Obviously the explicit expression for the energy of the 'electric' level $E^{(el)}$ is given by the right-hand side of equation (4.2).

Equation (5.3) describes the ground level in the donor well perturbed by the left-hand boundary of the QW. The solution to this transcendental equation can be found easily. In particular, for a wide QW for which $q_0(d+2b) \gg 1$, the expression for the donor level $W^{(0)}$ can be written in the form

$$W^{(0)}=W_0\left\{1-2\exp\left[-2q_0\left(\frac{d}{2}+b\right)\right]\right\}. \tag{5.4}$$

Thus, in the zeroth approximation, the system of the energy levels is the sum of two independent energies $E^{(el)}$ and $W^{(0)}$. For an arbitrary electric field F, the donor and 'electric' states are not in resonance. The electron having the energy $W^{(0)}$ is localized within the donor well whilst the electron having the energy $E^{(el)}$ is localized within the 'electric' well. Under the condition $W^{(0)}=E^{(el)}$ the donor and 'electric' levels appear to be in resonance. Using this condition and the expressions (4.2) and (5.4) for the energy levels, we arrive at the equation for the resonant electric field F_r for the wide QW:

$$-eF_r\left(\frac{d}{2}-b\right)+2.34E_0(F_r)=W_0\left\{1-2\exp\left[-2q_0\left(\frac{d}{2}+b\right)\right]\right\}. \tag{5.5}$$

In the first approximation, the last term on the left-hand side of equation (5.1) is taken into account. We expand the Airy function $\mathrm{Ai}(-\eta_2)$ and the second factor in the first term in (5.1) in power series in $q-q_2$ and $q-q_1$ respectively, where q_2 and q_1 are obtained in the zeroth approximation from equations (5.2) and (5.3). The second term is taken at $q=q_{1,2}$. Substituting the resulting expansions into equation (5.1) and using equations (5.2) and (5.3),

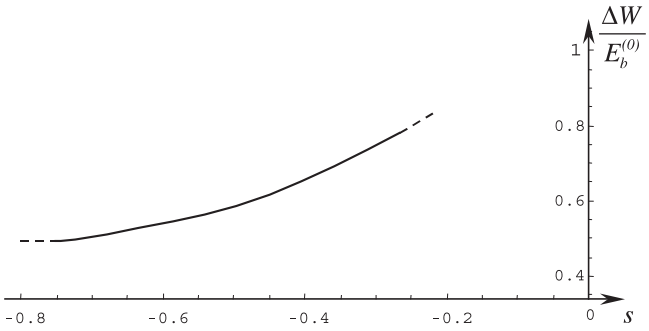

Figure 7. The dependence of the resonance gap ΔW (equation (5.8)) between the donor ($W^{(0)}$) and 'electric' ($E^{(el)}$ levels (see figure 1) on the dimensionless displacement $s = 2b/d$ of the impurity from the mid-point of a GaAs QW of width $d = 700$ Å; $E_b^{(0)}$ is the binding energy of the D$^-$ in bulk crystal.

we arrive at a quadratic equation for the parameter q. The roots of this equation $q^{(1,2)}$ can be written in the form

$$q^{(1,2)} = \tfrac{1}{2}(q_1 + q_2) \pm \left[\tfrac{1}{4}(q_1 - q_2)^2 + \Delta^2\right]^{1/2} \tag{5.6}$$

where

$$\Delta^2 = -\frac{q_1^2}{q_0 q_2}\left(\frac{\mu e F}{4\hbar^2}\right)^{2/3}\frac{\mathrm{Bi}(-2.34)}{\mathrm{Ai}'(-2.34)}\left[1 + 2(q_1 - q_0)\left(\frac{d}{2} + b\right)\right]^{-1}\exp(-2\Phi). \tag{5.7}$$

Equation (5.7) describes the effect of an anticrossing of the energy levels, which are derived from the states, which were originally located in the donor and 'electric' wells. It follows from (5.7) that, if the electric field F and the resonance value F_r are far apart, and hence $1/4(q_1 - q_2)^2 \gg \Delta^2$, the parameters $q^{(1,2)}$ are close to those obtained in the zeroth approximation, i.e. $q(1) \approx q_1$ and $q(2) \approx q_2$. In the case of resonance for which $F = F_r$ and $q_1 = q_2 \equiv q^{(0)}$, it is found that $q^{(1,2)} = q^{(0)} \pm \Delta$.

At resonance, the differences between the parameters $q^{(1)}$ and $q^{(2)}$ and their associated energies $W^{(2)} - W^{(1)} = \Delta W$ are given by

$$q^{(1)} - q^{(2)} = 2\Delta \qquad \text{and} \qquad \Delta W = \frac{2\hbar^2}{\mu}q^{(0)}\Delta. \tag{5.8}$$

Thus if resonance between the donor level (5.4) and the 'electric' level (4.2) occurs, crossing occurring in the zeroth approximation turns into anticrossing in the first approximation.

For a wide QW ($q_0 d \gg 1$, $q_1 \approx q_0$) the energy gap ΔW (equation (5.8)) becomes

$$\Delta W = 1.60|W_0|\left(\frac{F_r}{F_0}\right)^{1/3}\exp\left(-\frac{2F_0}{3F_r}\right) \tag{5.9}$$

where F_0 as well as above is the effective electric field associated with the charged donor and F_r is the resonant electric field. The dependence of the resonance gap ΔW on the position of the impurity b is depicted in figure 7.

6. Discussion

The basic physics associated with the behaviour of the binding energy of a charged donor in parallel fields remained transparent within our analysis. From the analytical approach described above, the dependences of the binding energy on the magnitudes of the external field, on the

width of the QW and on the position of the impurity have been obtained. Since the effects of the magnetic field and the width of the QW have been studied in detail previously (see [4] and references therein), we concentrate here on the influence of the electric field strength and the position of the impurity.

If the applied weak electric field $F < F_0$ and resonant field F_r are very different in magnitude, the system of the energy levels is the sum of the independent donor level W (equation (3.2)) and 'electric' level $E^{(el)}$ (equation (4.2)). The wavefunction is localized within the donor or 'electric' well respectively. The shift of the donor level $\Delta W(F, b)$ (equation (3.4)) depends on the applied electric field F and the effective field $F_1 \sim s$ (equation (3.6)) associated with the displacement $s = 2b/d$. The effects of both fields are described in section 3. The dependence of the binding energy E_b (equation (2.16)) of the D^- in the GaAs QW of width $d = 320$ Å on the electric field F is shown in figure 2. It is seen that the binding energy increases with increasing electric field. The reason for this is that for a weak electric field the energy red-shift of the free electron in the QW $\Delta E^{(0)} \approx -\mu e^2 F^2 d^4 (24\pi\hbar^2)^{-1}$ (equation (3.10)) is less than that of the weakly bound extra electron $\Delta W \approx -\mu e^2 F^2 (\hbar^2 q_1^4)^{-1}$ (equation (3.8)) for the QW of width $d = 320$ Å for which the parameter $q_1 d \approx 1.8$ and the donor field $F_0 \approx 9 \times 10^4$ V m^{-1}. However, for the neutral donor having a significantly greater binding energy $\sim q_1^2$, the relationship between the above-mentioned energy red-shifts may be different. Using the data presented in [19, 22] for the D^0 in the GaAs QW of width $d \approx 320$ Å in the presence of a strong magnetic field providing the parameter $\gamma = 5$, we estimate $\Delta W/\Delta E^{(0)} \approx 0.05$. In the absence of the magnetic field the binding energy of the D^0 becomes less. For a wide QW of width $d \gg a_0$, the energy red-shift ΔW can be estimated as $\Delta W = -9\mu e^2 F^2 a_0^4 (4\hbar^2)^{-1}$. For a GaAs QW ($a_0 = 98.7$ Å of width $d = 500$ Å we obtain $\Delta W/\Delta E^{(0)} \approx 0.5$. Thus the binding energy of the D^0 decreases with increasing electric field such that $\Delta E_b(F) \sim -F^2$. This is in agreement with the results of the variational calculations performed by Yoo *et al* [23] and Latge *et al* [24]. One can see in figure 2 that as the impurity centre moves away from the mid-point of the QW, the effect of the electric field on the binding energy of the D^- becomes more pronounced. The reason for this is that the extra electron is found to be less strongly bound (see figure 3).

The binding energy E_b (equation (2.16)) as a function of the displacement b in the presence of a weak electric field is presented in figure 3. Generally this dependence has an asymmetric form. The binding energy reaches a maximum for a certain position b of the impurity determined by the electric field. In the absence of the electric field ($F = 0$) the binding energy has a maximum for the impurity positioned at the mid-point of the QW ($b = 0$). With increasing electric field strength F the maximum is shifted insignificantly towards the right-hand boundary of the QW following the displacement of the electron density.

It follows from equation (3.11) and figure 4 that for weak electric field $F < F_0$ the inversion shift of the binding energy δE_b is proportional to both the electric field strength F and the position of the impurity b. The greater each of these parameters is, the greater the inversion shift δE_b is. If the width of the QW increases, the inversion shift decreases and vanishes in the limiting case of bulk material.

In the case of resonance for which $F \approx F_r$, the donor and 'electric' states become very close in energy. The resonant gap ΔW is defined by equations (5.8) and (5.9). On ignoring possible relaxation processes, coherent resonant tunnelling between the donor well and triangular well becomes possible. As a result, a drastic redistribution of the wavefunction and consequent emission of high-frequency coherent radiation occurs. The wavefunctions related to the components of the energy doublet attain a twin-peak configuration.

It is clear from equations (5.8), (5.7) and (5.9) that the resonant gap ΔW increases as the resonant field F_r increases. Equation (5.5) enables the dependence of the resonant field

F_r on the width of the QW d and the position of the impurity centre b to be obtained. The wider the QW, the smaller the resonant field F_r. The shift of the impurity centre towards the right-hand boundary (b increases) leads to an increase of the resonant field F_r. Figure 7 shows the dependence of the resonant gap ΔW as a function of the position of the impurity b.

For a strong electric field $F > F_0$, the binding energy $E_b = -\Delta W$, where ΔW (equation (4.3)) is the shift of the energy associated with the extra electron captured by the neutral donor. The binding energy $E_b(b)$ (see figure 5) follows the electron density $|\varphi(z)|^2$, where $\varphi(z)$ is the wavefunction of the extra electron in the QW in the presence of the electric field F. For the impurity positioned close to the right boundary of the QW ($b \approx d/2, s \approx 1$), $\varphi(z) \sim \mathrm{Ai}(-\eta_2) \approx 0$ and $E_b \approx 0$. When the impurity centre is shifted to the left-hand boundary of the QW, both the electron density and the binding energy increase and each reaches a maximum at $z = b = d/2 - 1.34\sigma^{-1}$. Further shift in the same direction leads to the binding energy decreasing. The relative binding energy E_b/E_0 decreases with increasing electric field for the impurity positioned at about $s < 0.45$ and increases for $s > 0.45$. Following the notation used in figure 5, we have $E_{b1} = 1.16$ meV, $E_{b2} = 0.78$ meV and $E_{b3} = 0.47$ meV for the position $s = -0.5$, while for the position $s = 0.6$ we obtain $E_{b1} = 0.65$ meV, $E_{b2} = 1.06$ meV, $E_{b3} = 1.48$ meV. This behaviour is explained by the position of the impurity relative to the electron density maximum. The electron density is shifted towards the right-hand boundary with increasing electric field strength. As a result, the binding energy decreases if the impurity is positioned in the left-hand side of the QW because the electric field separates the extra electron and the impurity centre. For the case of the impurity located close to the right-hand boundary, the electric field brings the extra electron closer to the impurity and the binding energy increases. The above physics is common to both charged and neutral donors. For this reason the reflected (figure 5) dependences of the binding energy of the D^- on the electric field strength F and position of the impurity b qualitatively coincide with those obtained numerically in [25–27] in which the corresponding dependences were thoroughly studied for the D^0.

As pointed out above, for a weak electric field $F < F_0$, the inversion shift δE_b (equation (3.11)) is a linearly increasing function of the position of the impurity b. In contrast to this case, for a strong electric field $F > F_0$, the inversion shift (4.6) is a nonmonotonic function of the position b and reaches a maximum for the impurity positioned away from the boundaries of the QW (figure 6). The reason for this is that for the impurity positions $b \approx \pm d/2$, the binding energy $E_b = -\Delta W$ (equation (4.3)) is a small quantity, while for the impurity positioned in the intermediate region of the QW, the binding energy becomes larger (see figure 5).

Let us consider possible experiments. Suitable values for the parameters for the GaAs QW are needed for the case of a strong magnetic field. Thus we take $\mu = 0.067m_0$, $\varepsilon = 12.5$, $a_0 = 98.7$ Å and $R_y = 5.83$ meV with $B = 33.5$ T and $\gamma = (a_0/a_B)^2 = 5$. The binding energy $E_b(0) = |W_0|$ in bulk material subjected to the chosen magnetic field was calculated in [1, 28], giving for $\gamma = 5$ the result $E_b(0) = 0.591R_y = 3.45$ meV. For the impurity centre positioned at $b \approx -0.5(d/2)$ in a QW of width $d \approx 700$ Å, the resonant splitting ΔW is defined by (5.9) such that $\Delta W \approx 2.1$ meV. This gap corresponds to a frequency of 0.50 THz for the emitted radiation. The chosen width of the QW causes the resonant electric field F_r and consequently penetration through the potential barrier to be relatively weak. This allows us to use our analytical approach. When the QW becomes narrower, the resonant field F_r and penetration increase and the above method of solving equation (2.12) becomes inappropriate. However, clearly in the presence of stronger electric fields the effect of the resonant splitting holds for the QWs of standard width. In this case a numerical calculation should be used.

For QWs of widths typically used in experiments the parameter q_1 can be found from equation (3.3). For a QW of width $d = 320$ Å we obtain for the energy $-W(0,0) = \frac{\hbar^2 q_1^2}{2\mu} \approx 1.67$ meV with the corresponding donor electric field $F_0 = 9.0 \times 10^4$ V m^{-1}. The inversion shift δE_b (equation (3.11)) produced by the displacement of the impurity $b \approx 96$ Å and relatively weak electric field $F \approx 4 \times 10^4$ V m^{-1} is $\delta E_b \approx 0.35$ meV. With increase of the electric field F, the inversion shift increases. For the electric field $F \approx 9 \times 10^5$ V m^{-1} and the same displacement, equation (4.5) gives for the inversion shift $\delta E_b \approx 1.2$ meV, which can be detected in experiments.

In spite of the fact that the approximation of infinitely high barriers is extensively used for the wide ($d > a_0$) QWs, we are aware that, for a detailed quantitative comparison with experimental results, barriers with a finite height V_0 have to be studied. In particular, for a weak electric field, the $E^{(0)}$ in equation (2.16) becomes

$$E^{(0)} = \frac{\hbar^2 \pi^2}{2\mu d^2}\left(1 - \frac{4\hbar}{d\sqrt{2\mu V_0}}\right). \tag{6.1}$$

For the GaAs QW of width $d = 320$ Å, a finite barrier of height $V_0 = 35R_y$ reduces the binding energy by about $0.2E^{(0)}$ and yields a discrepancy compared to results of variational calculations of the order of several per cent that is of the same order as those obtained by various numerical methods [18]. However, the finite height V_0 of the barrier does not introduce any novel physics and effects into the problem of a D^- in a QW.

Although magnetic fields beyond 30 T are created typically in terms of pulses, time-independent fields of the same strength are available and can be applied to study the optical response of low-dimensional structures [14]. Employing the adiabatic approximation implies the application of magnetic field strengths $B \geqslant 30$ T which requires significant experimental effort. Nevertheless, we believe that the current advances as regards the experimental availability of strong magnetic fields is a justification for studying semiconductor nanostructures subjected to strong external fields.

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